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Scalable Implementation of Multi-qubit Quantum Grover Search with Atomic Ensembles by Adiabatic Passage

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Abstract We propose a simple scheme to implement multi-qubit quantum Grover search with atomic ensembles by adiabatic passage. The scheme is immune to the atomic spontaneous emission, cavity decay and fiber decay. Furthermore, the process can be speeded up with atomic ensemble instead of single atom, which is important in view of decoherence. With each atomic ensemble confined in an individual cavity, our scheme is experimentally scalable to multi-qubit cases.

Keywords Atomic ensemble · Grover search · Adiabatic passage

The quantum Grover search algorithm [1, 2] is an efficient algorithm to look for one item in an unsorted database. Its efficiency has been verified experimentally in few-qubit cases in the context of NMR [3, 4], trapped ions [5], and optics [6, 7]. Theoretically, there are also many works contributing either to the algorithm itself [8, 9], or to its realization with various physical systems [10–16].

The algorithm can be briefly described as follows: For items represented by the computational states X with $X = 0, 1, \dots, V-1$, in a quantum register with M qubits, we have $V = 2^M$ possible states. It starts from a superposition state $|\Psi_0\rangle = (1/\sqrt{V})\sum_{X=0}^{V-1}|X\rangle$. One search step (i.e., a query) includes two operations [17]: i) inverting the amplitude of the target item $|\tau\rangle$, i.e., implementing the conditional phase gate [18–21] $I_\tau = I - 2|\tau\rangle\langle\tau|$, where I is the $V \times V$ identity matrix; ii) performing a diffusion transform D, i.e., inversion about

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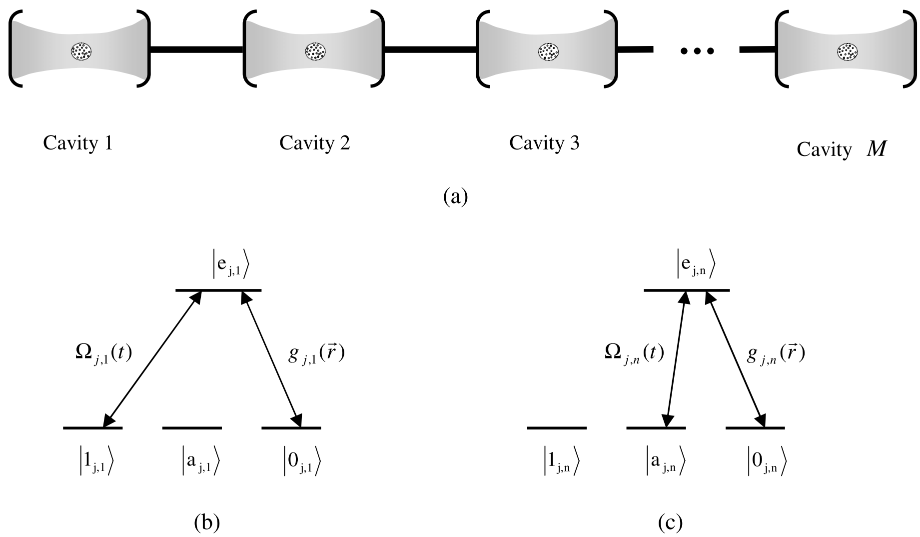

Fig. 1 (**a**) M cavities arrayed in a line are connected by $M-1$ fibers. And in each of the cavities, there is an atomic ensemble inside. (**b**) Interaction configuration for atoms in cavity 1. (**c**) Interaction configuration for atoms in cavity n ($n = 2, 3, \ldots, M$)

the average state $|\Psi_0\rangle$ with $D_{\mu\nu} = 2/V$ for $\mu \neq \nu$ and $D_{\mu\mu} = -1 + 2/V$. After $O(\sqrt{V})$ queries, the amplitude of the identified target will be increased while the amplitudes of non-target items are decreased. As a result, we can get the target item with high probability by measurement.

Cavity QED, as one of the potential candidates for quantum computation due to its good coherence, however, is not scalable. Because adding a new qubit to the same cavity means that new decoherence channels can be introduced [22]. Moreover, the qubits in the same cavity must be well separated for individual addressing. With more qubits involved, the experimental difficulties are enhanced. One possible solution [22] to this scaling problem is to perform distributed quantum computing, in which each qubit is confined in an individual cavity. On the other hand, atomic ensemble qubits have attracted much attention due to their enhanced atom-photon interaction [23–26], which can relax the demanding requirement for strong coupling between single atom and single photon. In this paper, we present a simple scheme to realize the multi-qubit quantum Grover search with atomic ensembles, with each ensemble embedded in an individual cavity and the nearest-neighboring cavities connected by optical fibers. Our scheme has the following advantages: (i) It is experimentally scalable to many-qubit cases by confining each qubit in an individual cavity; (ii) Due to the absence of excited atomic states and non-zero photon field states in the dark states used for evolution, it is immune to the atomic spontaneous emission, cavity decay and fiber decay; (iii) With atomic ensemble qubits, the effective atom-cavity coupling strength is amplified, so the process can be speeded up, which is important in view of decoherence.

The key element in implementing the quantum Grover search is to realize the multi-qubit conditional phase gate. Let us consider a one-dimensional fiber-coupled cavity chain with M cavities, each of which has an atomic ensemble inside, as shown in Fig. 1 (a). We assume that all the atoms have an identical four-level tripod-type energy structure as depicted in Fig. 1 (b) and (c), and all the cavities have the same mode frequency. For atom j in cavity 1, the laser pulse with Rabi frequency $\Omega_{j,1}(t)$ resonantly couples level $|1_{j,1}\rangle$ and level $|e_{j,1}\rangle$.

While for atoms in other cavities, the laser pulse with Rabi frequency $\Omega_{j,n}(t)$ resonantly couples level $|a_{j,n}\rangle$ and level $|e_{j,n}\rangle$ $(n \neq 1)$. For all the atoms, level $|0_{j,n}\rangle$ and level $|e_{j,n}\rangle$ are resonantly coupled by the cavity field with coupling strength $g_{j,n}(\overrightarrow{r})$, which depends on atom j's position in cavity n. Under the rotating wave approximation, the Hamiltonian for the whole system in the interaction picture is

$$\begin{aligned} H_I &= \sum_{n=1}^{M} H_{AC}^{n} + \sum_{m=1}^{M-1} H_{CF}^{m} \\ H_{AC}^{1} &= \sum_{j=1}^{N} (\Omega_{j,1}(t)|e_{j,1}\rangle\langle 1_{j,1}| + g_{j,1}(\overrightarrow{r}) a_1 |e_{j,1}\rangle\langle 0_{j,1}|) + \text{H.C.} \\ H_{AC}^{n} &= \sum_{j=1}^{N} (\Omega_{j,n}(t)|e_{j,n}\rangle\langle a_{j,n}| + g_{j,n}(\overrightarrow{r}) a_n |e_{j,n}\rangle\langle 0_{j,n}|) + \text{H.C.} \quad (n \neq 1) \\ H_{CF}^{m} &= v_{2m-1} a_m^{\dagger} b_m + v_{2m} a_{m+1}^{\dagger} b_m + \text{H.C.} \end{aligned} \tag{1}$$

where a_n, a_m, b_m are annihilation operators for the field modes in cavity n, m and in fiber m respectively, v_{2m-1} (v_{2m}) is the coupling strength between fiber m and cavity m $(m + 1)$. In the above expression, we have assumed all the atomic ensembles have N atoms for simplicity and used the short fiber limit to get H_{CF}^{m} [27]. The qubit states of atomic ensemble in cavity n consist of collective ground state and collective singly excited state, i.e., $|0\rangle_n = |0_{1,n}\rangle|0_{2,n}\rangle \cdots |0_{j,n}\rangle \cdots |0_{N,n}\rangle$, $|1\rangle_n = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |0_{1,n}\rangle|0_{2,n}\rangle \cdots |1_{j,n}\rangle \cdots |0_{N,n}\rangle$.

We assume that initially all the cavity modes and fiber modes are in the vacuum states. If qubit 1 is in the collective ground state $|0\rangle_1$, the whole system would not be affected by the Hamiltonian. If qubit 1 is in the collective singly excited state $|1\rangle_1$, it is coupled to states $|e_{1,1}\rangle$, $|e_{2,1}\rangle$, $\ldots$, $|e_{j,1}\rangle$, $\ldots$, $|e_{N,1}\rangle$ with coupling strengths $\Omega_{1,1}(t)/\sqrt{N}$, $\Omega_{2,1}(t)/\sqrt{N}$, $\ldots$, $\Omega_{j,1}(t)/\sqrt{N}$, $\ldots$, $\Omega_{N,1}(t)/\sqrt{N}$ respectively. Then the cavity mode in cavity 1 couples all these states to $|0\rangle_1|1\rangle_{c1}$ ($|1\rangle_{c1}$ denotes the single-photon state in cavity 1) with the coupling strengths $g_{1,1}(\overrightarrow{r})$, $g_{2,1}(\overrightarrow{r})$, $\ldots$, $g_{j,1}(\overrightarrow{r})$, $\ldots$, $g_{N,1}(\overrightarrow{r})$ respectively. If the laser pulse is added collinearly with the cavity axis, then the pulse felt by the atom $\widetilde{\Omega}_{j,1}(t)$ has approximately the same spatial mode as the cavity mode [28, 29], i.e., $\widetilde{\Omega}_{j,1}(t) \sim f_1(t) g_{j,1}(\overrightarrow{r})$ $(j = 1, 2, \ldots, N)$. With this condition, we can simplify these transitions to be a three-level Λ-type configuration by Morris-Shore (MS) transformation [30, 31]. The effective Hamiltonian with respect to the transition from $|1\rangle_1|0\rangle_{c1}$ to $|0\rangle_1|1\rangle_{c1}$ can be written as $H_{AC}^{1} = \Omega_1 |\xi_1\rangle|0\rangle_{c1c1}\langle 0|_1\langle 1| + G_1 |\xi_1\rangle|0\rangle_{c1c1}\langle 1|_1\langle 0| + \text{H.C.}$, where $G_1 = \sqrt{\sum_{j=1}^{N} g_{j,1}^2(\overrightarrow{r})}$, $\Omega_1 \sim f_1(t) G_1/\sqrt{N}$, $|\xi_1\rangle = \frac{1}{G_1} \sum_{j=1}^{N} g_{j,1}(\overrightarrow{r})|e_{j,1}\rangle$. The photon in cavity 1 would be transmitted to other cavities by optical fibers. If the qubit in cavity n is in state $|0\rangle_n$, it would absorb the photon. Similarly, this ensemble can be addressed with the laser pulse collinearly with the cavity axis. So the MS transformation can also be applied. The effective Hamiltonian with respect to the transition from $|0\rangle_n|1\rangle_{cn}$ to $|a\rangle_n|0\rangle_{cn}$ can be written as $H_{AC}^{n} = \Omega_n |\xi_n\rangle|0\rangle_{cncn}\langle 0|_n\langle a| + G_n |\xi_n\rangle|0\rangle_{cncn}\langle 1|_n\langle 0| + \text{H.C.}$, where $G_n = \sqrt{\sum_{j=1}^{N} g_{j,n}^2(\overrightarrow{r})}$, $\Omega_n \sim f_n(t) G_n/\sqrt{N}$, $|\xi_n\rangle = \frac{1}{G_n} \sum_{j=1}^{N} g_{j,n}(\overrightarrow{r})|e_{j,n}\rangle$, $|a\rangle_n = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |0_{1,n}\rangle|0_{2,n}\rangle \cdots |a_{j,n}\rangle \cdots |0_{N,n}\rangle$. For simplicity, we denote all the states in the following by omitting the zero states in the cavity modes and fiber modes. For clarity, we begin with $M = 3$. States with qubit 1 in state $|0\rangle_1$ are not affected in the process. The dark states with qubit 1 in state $|1\rangle_1$ can be calculated as follows:

$$
\begin{aligned}
D_{111} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |1\rangle_2 |1\rangle_3 - |0\rangle_1 |1\rangle_2 |1\rangle_3 (v_2 v_4 |1\rangle_{c1} - v_1 v_4 |1\rangle_{c2} + v_1 v_3 |1\rangle_{c3}) \\
D_{110} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |1\rangle_2 |0\rangle_3 - |0\rangle_1 |1\rangle_2 |0\rangle_3 (v_2 v_4 |1\rangle_{c1} - v_1 v_4 |1\rangle_{c2} + v_1 v_3 |1\rangle_{c3}) \\
&\quad + \frac{G_3}{\Omega_3} v_1 v_3 |0\rangle_1 |1\rangle_2 |a\rangle_3 \\
D_{101} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |0\rangle_2 |1\rangle_3 - |0\rangle_1 |0\rangle_2 |1\rangle_3 (v_2 v_4 |1\rangle_{c1} - v_1 v_4 |1\rangle_{c2} + v_1 v_3 |1\rangle_{c3}) \\
&\quad - \frac{G_2}{\Omega_2} v_1 v_4 |0\rangle_1 |a\rangle_2 |1\rangle_3 \\
D_{100} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |0\rangle_2 |0\rangle_3 - |0\rangle_1 |0\rangle_2 |0\rangle_3 (v_2 v_4 |1\rangle_{c1} - v_1 v_4 |1\rangle_{c2} + v_1 v_3 |1\rangle_{c3}) \\
&\quad - \frac{G_2}{\Omega_2} v_1 v_4 |0\rangle_1 |a\rangle_2 |0\rangle_3 + \frac{G_3}{\Omega_3} v_1 v_3 |0\rangle_1 |0\rangle_2 |a\rangle_3
\end{aligned}
\tag{2}
$$

If $G_n \gg \Omega_n$ $(n = 1, 2, 3)$ are satisfied, the population of single-photon cavity states $|1\rangle_{cn}$ $(n = 1, 2, 3)$ can be neglected. Then the above dark states can be reduced to

$$
\begin{aligned}
D_{111} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |1\rangle_2 |1\rangle_3 \\
D_{110} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |1\rangle_2 |0\rangle_3 + \frac{G_3}{\Omega_3} v_1 v_3 |0\rangle_1 |1\rangle_2 |a\rangle_3 \\
D_{101} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |0\rangle_2 |1\rangle_3 - \frac{G_2}{\Omega_2} v_1 v_4 |0\rangle_1 |a\rangle_2 |1\rangle_3 \\
D_{100} &\propto \frac{G_1}{\Omega_1} v_2 v_4 |1\rangle_1 |0\rangle_2 |0\rangle_3 - \frac{G_2}{\Omega_2} v_1 v_4 |0\rangle_1 |a\rangle_2 |0\rangle_3 + \frac{G_3}{\Omega_3} v_1 v_3 |0\rangle_1 |0\rangle_2 |a\rangle_3
\end{aligned}
\tag{3}
$$

Initially, $\Omega_1 \ll \Omega_2, \Omega_3$. By adiabatically increasing Ω_1 and decreasing Ω_2, Ω_3 to satisfy $\Omega_1 \gg \Omega_2, \Omega_3$, we have (assuming all the pulses having the same phase)

$$
\begin{aligned}
&|1\rangle_1 |1\rangle_2 |1\rangle_3 \to |1\rangle_1 |1\rangle_2 |1\rangle_3 \\
&|1\rangle_1 |1\rangle_2 |0\rangle_3 \to |0\rangle_1 |1\rangle_2 |a\rangle_3 \\
&|1\rangle_1 |0\rangle_2 |1\rangle_3 \to -|0\rangle_1 |a\rangle_2 |1\rangle_3 \\
&|1\rangle_1 |0\rangle_2 |0\rangle_3 \to -\frac{G_2}{\Omega_2} v_1 v_4 |0\rangle_1 |a\rangle_2 |0\rangle_3 + \frac{G_3}{\Omega_3} v_1 v_3 |0\rangle_1 |0\rangle_2 |a\rangle_3
\end{aligned}
\tag{4}
$$

Then we reverse the above process but with phases regarding Ω_2 and Ω_3 added by π, i.e., $\Omega_2 \to -\Omega_2$, $\Omega_3 \to -\Omega_3$, the states would become

$$
\begin{aligned}
&|1\rangle_1 |1\rangle_2 |1\rangle_3 \to |1\rangle_1 |1\rangle_2 |1\rangle_3 \\
&|0\rangle_1 |1\rangle_2 |a\rangle_3 \to -|1\rangle_1 |1\rangle_2 |0\rangle_3 \\
&-|0\rangle_1 |a\rangle_2 |1\rangle_3 \to -|1\rangle_1 |0\rangle_2 |1\rangle_3
\end{aligned}
\tag{5}
$$

$$-\frac{G_2}{\Omega_2}v_1v_4|0\rangle_1|a\rangle_2|0\rangle_3+\frac{G_3}{\Omega_3}v_1v_3|0\rangle_1|0\rangle_2|a\rangle_3\rightarrow -|1\rangle_1|0\rangle_2|0\rangle_3$$

After a single-qubit phase gate $\sigma_{z,1}=|0\rangle_{11}\langle 0|-|1\rangle_{11}\langle 1|$ on qubit 1, and considering that the states $|0\rangle_1|1\rangle_2|1\rangle_3$, $|0\rangle_1|1\rangle_2|0\rangle_3$, $|0\rangle_1|0\rangle_2|1\rangle_3$, $|0\rangle_1|0\rangle_2|0\rangle_3$ are not affected, we can get the three-qubit conditional phase gate $I_{|1\rangle_1|1\rangle_2|1\rangle_3}$, which is to label the target state $|1\rangle_1|1\rangle_2|1\rangle_3$. It is straightforward to extend the method to implement M-qubit conditional phase gate. If $G_n\gg\Omega_n$ ($n=1,2,\ldots,M$) are satisfied, the dark states with qubit 1 in state $|1\rangle_1$ are given as follows:

$$\begin{aligned}
D_{11\ldots 11}&\propto\frac{G_1}{\Omega_1}\prod_{m=1}^{M-1}v_{2m}|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M\\
D_{11\ldots 10}&\propto\frac{G_1}{\Omega_1}\prod_{m=1}^{M-1}v_{2m}|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M\\
&\quad+(-1)^{M+1}\frac{G_M}{\Omega_M}\prod_{m=1}^{M-1}v_{2m-1}|0\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|a\rangle_M\\
D_{11\ldots 01}&\propto\frac{G_1}{\Omega_1}\prod_{m=1}^{M-1}v_{2m}|1\rangle_1|1\rangle_2\cdots|0\rangle_{M-1}|1\rangle_M\\
&\quad+(-1)^{M}\frac{G_{M-1}}{\Omega_{M-1}}\prod_{m=1}^{M-2}v_{2m-1}v_{2(M-1)}|0\rangle_1|1\rangle_2\cdots|a\rangle_{M-1}|1\rangle_M\\
&\cdots\\
D_{10\ldots 00}&\propto\frac{G_1}{\Omega_1}\prod_{m=1}^{M-1}v_{2m}|1\rangle_1|0\rangle_2\cdots|0\rangle_{M-1}|0\rangle_M\\
&\quad+\sum_{n=2}^{M}(-1)^{n+1}\frac{G_n}{\Omega_n}\prod_{m=1}^{n-1}v_{2m-1}\prod_{m=n}^{M-1}v_{2m}|0\rangle_1|0\rangle_2\cdots|a\rangle_n\cdots|0\rangle_{M-1}|0\rangle_M
\end{aligned}\tag{6}$$

Similar to the three-qubit case, the procedure includes three steps: 1) Initially, $\Omega_1\ll\Omega_n$ ($n=2,3,\ldots,M$). By adiabatically increasing Ω_1 and decreasing Ω_n ($n=2,3,\ldots,M$) to satisfy $\Omega_1\gg\Omega_n$ ($n=2,3,\ldots,M$), the populations of all computational states except $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M$ are transferred to some intermediate states; 2) Then we reverse the adiabatic evolution in step 1 but with Ω_n ($n=2,3,\ldots,M$) changed to be $-\Omega_n$. In this way, all populations in the intermediate states are transferred back to the corresponding computational states but with a π phase added. After this step, all the computational states with qubit 1 in state $|1\rangle_1$ except $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M$ are added by a minus sign; 3) Finally, the single-qubit phase gate $\sigma_{z,1}=|0\rangle_{11}\langle 0|-|1\rangle_{11}\langle 1|$ on qubit 1 adds all the computational states in expression (6) with a π phase. During the whole process, computational states with qubit 1 in state $|0\rangle_1$ could never have an evolution. So to sum it all up, we can get an M-qubit conditional phase gate $I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M}$, which is to label the target state $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M$.

All the other conditional phase gates in the M-qubit Grover search can be achieved by adding NOT gates $\sigma_{x,n}$ ($n=1,2,\ldots,M$) on both sides of $I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M}$, where $\sigma_{x,n}$ is the NOT gate acting on qubit n. For instance, $I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M}=\sigma_{x,M}I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M}\sigma_{x,M}$,

$I_{|0\rangle_1|0\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M} = \sigma_{x,2}\sigma_{x,1} I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M} \sigma_{x,1}\sigma_{x,2}$. Diffusion transform D can also be obtained by $\prod_{n=1}^{M} H_n I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M} \prod_{n=1}^{M} H_n$, where H_n is the Hadamard gate acting on qubit n, transforming states as $|1\rangle_n \to (1/\sqrt{2})(|1\rangle_n + |0\rangle_n)$, $|0\rangle_n \to (1/\sqrt{2})(|1\rangle_n - |0\rangle_n)$. So once we get the multi-qubit conditional phase gate $I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M}$, we can get all the multi-qubit gates needed in the Grover search. Moreover, the single-qubit operations on atomic ensemble qubits can be easily obtained by the Rydberg blockade effect [32].

Now, we show explicitly the process for a M-qubit Grover search. Initially, all the qubits are in the collective singly excited states, i.e., $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M$. After the Hadamard gates on all the qubits, we can get

$$\prod_{n=1}^{M} H_n |1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M = (1/\sqrt{V})\sum_{X=0}^{V-1}|X\rangle = |\Psi_0\rangle \tag{7}$$

The search starts from the average state $|\Psi_0\rangle$, where each computational states has an equal probability to be picked up. One search step includes two operations, I_τ and D. If the target state is $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M$, one search step corresponds to the transformation

$$D I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M} = \prod_{n=1}^{M} H_n I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M} \prod_{n=1}^{M} H_n \sigma_{x,M} I_{|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|1\rangle_M} \sigma_{x,M}$$

$$= \begin{bmatrix} 1-2/V & -2/V & -2/V & \cdots & -2/V & -2/V \\ -2/V & 1-2/V & -2/V & \cdots & -2/V & -2/V \\ -2/V & -2/V & 1-2/V & \cdots & -2/V & -2/V \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -2/V & -2/V & -2/V & \cdots & 1-2/V & -2/V \\ -2/V & -2/V & -2/V & \cdots & -2/V & 1-2/V \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2/V & 2/V & -2/V & \cdots & -2/V & -2/V \\ -2/V & -1+2/V & -2/V & \cdots & -2/V & -2/V \\ -2/V & 2/V & 1-2/V & \cdots & -2/V & -2/V \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -2/V & 2/V & -2/V & \cdots & 1-2/V & -2/V \\ -2/V & 2/V & -2/V & \cdots & -2/V & 1-2/V \end{bmatrix}, \tag{8}$$

where the above are $V \times V$ matrices, and $V = 2^M$. After O($\sqrt{V}$) search steps, i.e., O($\sqrt{V}$) successive operations of expression (8) on average state $|\Psi_0\rangle$, the amplitude of target state $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M$ would be increased while the amplitudes of non-target states are negligible. Then by measurement, we can get the target state $|1\rangle_1|1\rangle_2\cdots|1\rangle_{M-1}|0\rangle_M$ with high probability.

In summary, we have proposed a simple scheme to realize multi-qubit quantum Grover search with atomic ensembles by adiabatic passage. It is easy to see that due to the absence of the atomic excited states or non-zero photon field states in the dark states in expression

(6), the adiabatic evolution along these states is immune to the atomic spontaneous emission, cavity decay and fiber decay. In fact, expression (6) can also be applied to the single atom case if we replace G_n with the single atom-cavity coupling strength g. Compared with the single atom case, G_n is larger than g, so the value for Ω_n can also be increased under the condition of $G_n \gg \Omega_n$. Meanwhile, the adiabatic condition requires $\Omega_n T \gg 1$ [33], where T is the pulse duration. So with increased Ω_n, the pulse duration can be shortened, which can reduce the probability of dissipation. In real experiments, the number of atoms in each cavity varies from one another. This has no effect on the qubit encoding, since the qubit is characterized by the collective ground state and the collective singly excited state, which has no relation with the number of atoms. But the fluctuation of N could affect the value of G_n, because G_n is related to the sum of $g^2_{j,n}(\overrightarrow{r})$ of all the atoms in the cavity. However, this has no effect on our scheme either. Because the equal G_n $(n = 1, 2, 3, \ldots, M)$ is not required. With each atomic ensemble embedded in an individual cavity, we believe our scheme is experimentally scalable to multi-qubit cases.

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References

1. Grover, L.K.: Phys. Rev. Lett. **79**, 325 (1997)
2. Grover, L.K.: Phys. Rev. Lett. **79**, 4709 (1997)
3. Chuang, I.L., Gershenfeld, N., Kubinec, M.: Phys. Rev. Lett. **80**, 3408 (1998)
4. Jones, J.A., Mosca, M., Hansen, R.H.: Nature (London) **393**, 344 (1998)
5. Brickman, K.-A., Haljan, P.C., Lee, P.J., Acton, M., Deslauriers, L., Monroe, C.: Phys. Rev. A **72**, 050306(R) (2005)
6. Bhattacharya, N., van Linden van den Heuvell, H.B., Spreeuw, R.J.C.: Phys. Rev. Lett. **88**, 137901 (2002)
7. Walther, P., Resch, K.J., Rudolph, T., Schenck, E., Weinfurter, H., Vedral, V., Aspelmeyer, M., Zeilinger, A.: Nature (London) **434**, 169 (2005)
8. Farhi, E., Gutmann, S.: Phys. Rev. A **57**, 2403 (1998)
9. Pérez, A., Romanelli, A.: Phys. Rev. A **76**, 052318 (2007)
10. Scully, M.O., Zubairy, M.S.: Phys. Rev. A **65**, 052324 (2002)
11. Deng, Z.J., Feng, M., Gao, K.L.: Phys. Rev. A **72**, 034306 (2005)
12. Zheng, X.-H., Dong, P., Xue, Z.-Y., Cao, Z.-L.: Physica C **455**, 76 (2007)
13. Yang, W.L., Chen, C.Y., Feng, M.: Phys. Rev. A **76**, 054301 (2007)
14. Daems, D., Guérin, S.: Phys. Rev. Lett. **99**, 170503 (2007)
15. Ivanov, S.S., Ivanov, P.A., Vitanov, N.V.: Phys. Rev. A **78**, 030301(R) (2008)
16. Yang, W.-L., Wei, H., Chen, C.-Y., Feng, M.: J. Opt. Soc. Am. B **25**, 1720 (2008)
17. Feng, M.: Phys. Rev. A **63**, 052308 (2001)
18. Goto, H., Ichimura, K.: Phys. Rev. A **70**, 012305 (2004)
19. Xiao, Y.-F., Zou, X.-B., Guo, G.-C.: Phys. Rev. A **75**, 014302 (2007)
20. Zou, X.-B., Xiao, Y.-F., Li, S.-B., Yang, Y., Guo, G.-C.: Phys. Rev. A **75**, 064301 (2007)
21. Zhang, X.L., Feng, X.-L., Wu, C., Oh, C.H.: Phys. Rev. A **79**, 034301 (2009)
22. Barrett, S.D., Kok, P.: Phys. Rev. A **71**, 060310(R) (2005)
23. Duan, L.-M., Lukin, M.D., Cirac, J.I., Zoller, P.: Nature (London) **414**, 413 (2001)
24. Xiao, Y.-F., Zou, X.-B., Han, Z.-F., Guo, Guang-Can: Phys. Rev. A **74**, 044303 (2006)
25. Yin, Z.Q., Li, F.L.: Phys. Rev. A **75**, 012324 (2007)
26. Lin, G.-W., Zou, X.-B., Lin, X.-M., Guo, G.-Can: Phys. Rev. A **79**, 042332 (2009)
27. Serafini, A., Mancini, S., Bose, S.: Phys. Rev. Lett. **96**, 010503 (2006)
28. Duan, L.-M., Kuzmich, A., Kimble, H.J.: Phys. Rev. A **67**, 032305 (2003)
29. Duan, L.-M., Kimble, H.J.: Phys. Rev. Lett. **90**, 253601 (2003)
30. Kyoseva, E.S., Vitanov, N.V.: Phys. Rev. A **73**, 023420 (2006)
31. Xu, Z.Y., Hu, Y.M., Yang, W.L., Feng, M., Du, J.F.: Phys. Rev. A **80**, 022335 (2009)
32. Lukin, M.D., Fleischlaher, M., Cote, R., Duan, L.M., Jaksch, D., Cirac, J.I., Zoller, P.: Phys. Rev. Lett. **87**, 037901 (2001)
33. Bergmann, K., Theuer, H., Shore, B.W.: Rev. Mod. Phys. **70**, 1003 (1998)